Relations and Functions – Exercises

(**Solutions**)

*CS 5012*

1. Using the sets below of students and test scores (highest grade = 100), write out the expression, domain, range, and codomain.
2. Let *S* be the set of students at UVA, let *M* be the set of sporting events at UVA, and let *V(s,m)* be “student *s* has been to *m.*” Rewrite each of the following statements without using the symbol , and , or variables.
   1. such that *V(s, Basketball)*
   2. , *V(s, Football)*
   3. , such that *V(s,m)*
   4. such that , *V(s,m)*
3. For each of the following relations on the set {5, 6, 7, 8} decided whether it is reflexive, symmetric, and/or transitive. Which of these are equivalence relations?
   1. {(6,6), (6,7), (6,8), (7,6), (7,7), (7,8)}
   2. {(5,5), (5,6), (6,5), (6,6), (7,7), (8,8)}
   3. {(6,8), (8,6)}
4. Calculate the composition (g(f(x))) of the following two functions, f(x) and g(x):

f(x) = x + 5 g(x) = x2 + 2x + 10

1. *Answers*:
   * Expression:
   * Domain: {Bill, Joe, Daniel, Sam}
   * Range {96, 90, 99, 85}
   * Codomain {x | 0 < x < 100, x is only positive}
2. *Answers*:
   * a: There exists a student that has been to *Basketball*.
   * b: All students have been to *Football*.
   * c: All students have seen a sporting event.
   * d: There exists a sporting event that all students have seen.
3. *Answers*:
   * a: Not reflexive: (8,8) is missing. Not symmetric (6,8) belongs but (8,6) does not. Transitive.
   * b: Reflexive. Symmetric. Transitive.
   * c: Not reflexive: (6,6) and (8,8) are missing. Symmetric. Not transitive since (6,6) is not included.
4. *Answers*:
   * Replace ‘x’ in g(x) with (x+5):

(x+5)2 + 2(x+5) + 10

* + Solve: g(f(x))

= (x+5)(x+5) + 2x + 10 + 10

= x2 + 10x + 25 + 2x + 10 + 10

= x2 + 12x + 45